EX 1

After having defined what we intend for equilibrium of the dynamical system \( \dot{x} = f(x) \), consider

\[
f(x) = x(1-x)(x+1)
\]

and compute all the equilibrium points, discuss their stability and plot qualitatively the free motion of the state for initial conditions \( x_0 = -0.001 \) and \( x_0 = 0.001 \) and those of the corresponding linearized system (in the origin).

EX 2

Consider the closed loop system in the following figure where \( G(s) = \frac{1-s/30}{(1+s/3)^2} \). Design a linear controller \( R(s) \) such that

- \(|e_\infty| \leq 0.1\) when \( w(t) = \text{step}(t) \) and \( d(t) = 0 \);
- \( \phi_m \geq 50 \text{ deg}, \omega_c \geq 1 \)
- Attenuation of at least 30 dB when \( d(t) = \sin(\omega t), \omega \leq 0.1, w(t) = 0 \)

Finally, propose a sampling time consistent with the phase margin and the selected crossover frequency.
EX 3
With reference to a closed loop SISO control scheme, discuss whether it is possible (or not) to have zero steady state error for both a step reference and a step measurement disturbance.

EX 4
Consider the discrete time system $x_{k+1} = Ax_k + Bu_k$, $y_k = Cx_k$ with

$$
A = \begin{bmatrix}
-2 & -1 \\
1 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix} 1 & 0 \end{bmatrix}^T, C = \begin{bmatrix} 1 & 0 \end{bmatrix}
$$
and discuss whether it is possible to derive a state feedback controller by minimizing the following cost functional

$$
J = \sum_{k=0}^{+\infty} y_k^T y_k + u_k^T u_k + \Delta u_k^T \Delta u_k
$$

where $\Delta u_k = u_k - u_{k-1}$.

EX 5
Consider the following nonlinear system where $\Gamma(s) = \frac{100s(1 + 0.2s)}{(1 + s)(1 + 6s)(1 + 15s)}$ while $N$ represents a nonlinearity in the sector $[-0.2, 1]$. Discuss the absolute stability of the origin.