AUTOMATIC CONTROL

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Identification and control of an atomic force microscope (AFM)
Atomic force microscope

The second device we are going to study in this course is the atomic force microscope (AFM).

The 1986 Nobel Prize in Physics was shared by Gerd Binnig and Heinrich Rohrer for their design of the scanning tunneling microscope (STM).

The idea of the instrument is to bring an atomically sharp tip so close to a conducting surface that tunneling occurs. An image is obtained by traversing the tip across the sample and measuring the tunneling current as a function of tip position.
Atomic force microscope – cont’d

The invention of STM has stimulated the development of a family of instruments permitting visualization of surface structure at the nanometer scale, including the AFM.

Within the AFM, a sample is probed by a tip on a cantilever. An AFM can operate in two modes:

- *tapping mode*: the cantilever is vibrated, and the amplitude of vibration is controlled by feedback
- *contact mode*: the cantilever is in contact with the sample, and its bending is controlled by feedback
Atomic force microscope – cont’d

The tip (radius of 10nm) can be moved vertically (and horizontally) using a piezoelectric scanner. Its tilt depends on the topography of the surface and the position of the base, controlled by the piezo element.
Atomic force microscope – cont’d

The vertical motion of the scanner is modeled by two masses supported by a spring-damper. The upper mass represents half of the piezo plus the mass of the sample, the lower part represents the supporting structure.

The equations are as follows

\[ M \frac{d^2 p}{dt^2} + C \frac{dp}{dt} + K p = \begin{bmatrix} F \\ -F \end{bmatrix} \quad p = \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \]

where

\[ M = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad C = \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \quad K = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \]
Atomic force microscope – cont’d

Letting $\mathbf{x} = [p_1 \ p_1 \ p_2 \ \dot{p}_2]^T$ the state vector, in order to write the equations of the system in their canonical form, we can first write

$$\ddot{\mathbf{p}} = M^{-1} \left( \begin{bmatrix} F \\ -F \end{bmatrix} - C \dot{\mathbf{p}} - K \mathbf{p} \right)$$

and perform very trivial algebraic manipulation in order to obtain the corresponding canonical form

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= \frac{-k_1 x_1 - c_1 x_2 + k_1 x_3 + c_1 x_4 + F}{m_1} \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= \frac{k_1 x_1 + c_1 x_2 - (k_1 + k_2) x_3 - (c_1 + c_2) x_4 - F}{m_2}
\end{align*}$$
Atomic force microscope – cont’d

In terms of system matrices, we have

\[
A = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-\frac{k_1}{m_1} & -\frac{c_1}{m_1} & \frac{k_1}{m_1} & \frac{c_1}{m_1} \\
0 & 0 & 0 & 1 \\
\frac{k_1}{m_2} & \frac{c_1}{m_2} & -\frac{k_1+k_2}{m_2} & -\frac{c_1+c_2}{m_2}
\end{bmatrix} 
\]

\[
B = \begin{bmatrix}
0 \\
\frac{1}{m_1} \\
0 \\
\frac{1}{m_2}
\end{bmatrix}
\]

The corresponding characteristic polynomial is as follows

\[
\det (sI - A) = s^4 + \frac{c_1 m_1 + c_1 m_2 + c_2 m_1}{m_1 m_2} s^3 + \frac{k_1 m_1 + k_1 m_2 + k_2 m_1 + c_1 c_2}{m_1 m_2} s^2 + \frac{c_1 k_2 + c_2 k_1}{m_1 m_2} s + \frac{k_1 k_2}{m_1 m_2} =
\]

\[
= s^4 + \alpha_3 s^3 + \alpha_2 s^2 + \alpha_1 s + \alpha_0
\]
Atomic force microscope – cont’d

Applying the Routh’s criterion to study the stability of the system is quite prohibitive from a symbolic point of view.

However, under reasonable assumption on the system parameters, we may write the characteristic polynomial as follows

\[
\det (sI - A) = \left( s^2 + 2\xi_1 \omega_1 s + \omega_1^2 \right) \left( s^2 + 2\xi_3 \omega_3 s + \omega_3^2 \right)
\]

where

\[
\omega_1 \approx \sqrt{\frac{k_2}{m_1 + m_2}} \quad \omega_3 \approx \sqrt{\frac{(m_1 + m_2) k_1}{m_1 m_2}}
\]

and similarly for the (positive) damping parameters. The system is therefore asymptotically stable.
Atomic force microscope

For the ATM what we can measure (through the pair laser-photodiode) is the position of the top of the stack (tip), therefore let

$$\mathbf{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}$$

The corresponding transfer function is as follows

$$G(s) = \frac{s^2 + 2\omega_2\xi_2 s + \omega_2^2}{m_1 \left( s^2 + 2\omega_1\xi_1 s + \omega_1^2 \right) \left( s^2 + 2\omega_3\xi_3 s + \omega_3^2 \right)}$$

The system has two pairs of complex and conjugate poles and a pair of complex and conjugate zeros.

As the parameters (apart from masses) correspond to modeling assumption (e.g. the piezo actuator is a spring), we might not be able to assign them reasonable values!
Atomic force microscope – cont’d

What we can do in order to get some reasonable value for those parameters is to excite the system with a harmonic signal (e.g. a sine wave). Correspondingly, we can measure, for varying frequencies the ratio between the amplitude of the exerted force and the corresponding displacement.

As the system is linear and asymptotically stable, we can apply the frequency response theorem. After a transient, we expect the output to be a sine wave with same frequency but possibly different amplitude (and phase).

\[ u(t) = A \sin(\omega_0 t) \rightarrow G(s) \rightarrow y_R(t) = B \sin(\omega_0 t + \phi) \]

For a given frequency, what is the ratio between output and input amplitudes, i.e. B/A?
Atomic force microscope – cont’d

What we get is indeed a dotted magnitude Bode diagram.
Atomic force microscope – cont’d

What we can do now is to fit those points with a curve depending on our unknown parameters in order to estimate them.

\[ \omega_1 = 40.9^{25} \text{kHz}, \omega_2 = 41.6, \omega_3 = 120 \]

\[ \xi_1 = \xi_2 = 3\%, \xi_3 = 40\% \]
Design specifications

We have seen that the transfer function of the AFM can be estimated from data through suitable experiments. As a result, we obtained

\[
G(s) = \frac{s^2 + 2\omega_2\xi_2 s + \omega_2^2}{m_1 (s^2 + 2\omega_1\xi_1 s + \omega_1^2) (s^2 + 2\omega_3\xi_3 s + \omega_3^2)}
\]

where

\[
\begin{align*}
\omega_1 &= 40.9 \text{ Hz}, \omega_2 = 41.6, \omega_3 = 120 \\
\xi_1 &= \xi_2 = 3\%, \xi_3 = 40\%, m_1 = 0.01 \text{ kg}
\end{align*}
\]

Therefore, we will focus on

\[
G(s) = \frac{100s^2 + 1.568e06s + 6.832e12}{s^4 + 6.186e05s^3 + 6.438e11s^2 + 4.86e16s + 3.754e22}
\]
Design specifications – cont’d

As we have already seen, the Bode diagrams are as follows
Design specifications – cont’d

The resulting closed-loop system must satisfy the following requirements:

1. contained overshoot (≤ 10%) in response to a reference step signal
2. null steady state error for ramp references
3. Attenuation of feedforward disturbances on controlled output of 20 dB until 1 kHz
4. the fastest transient (as possible)
5. attenuation of measurement noise on controlled output of 100 dB after 1 MHz
SISO design for AFM

As for the static design, specification 2. requires to nullify the steady state error for a reference ramp signal, therefore we focus on

\[ E(s) = S(s) Y^0(s) \]

By applying the final value theorem (assuming we will be able to stabilize the closed-loop system!) we have

\[ 0 = e_\infty = \lim_{s \to 0} sE(s) = \lim_{s \to 0} sS(s) \frac{1}{s^2} = \lim_{s \to 0} \frac{s^{gL-1}}{\mu_L + s^{gL}} \]

Therefore, in order to have \( g_L - 1 \geq 1, \forall \mu_L > 0 \) we select a type-2 controller, i.e.

\[ R_1(s) = \frac{\mu_R}{s^2} \]
SISO design for AFM – cont’d

What we obtained so far is as follows (notice that we are not crossing the 0 dB axis!)
SISO design for AFM – cont’d

We consider now a different gain for the static part of the controller controller, which corresponds to ten times (20 dB) inverse of

\[ |L_1(j\omega_d)| \approx 4.6e^{-18} \quad \omega_d = 1 \ kHz \]

This way, which corresponds to the following static controller

\[ R_1(s) = \frac{2.169e18}{s^2} \]

we are enforcing requirement 3. regarding the attenuation of feedforward (actuation) disturbances.
SISO design for AFM – cont’d

The corresponding Bode diagrams are as follows

\[ |\hat{L}_1(j\omega)| = \mu_R |L_1(j\omega)| \]

- Attenuation of 20 dB of \(d(t)\)
- Closed-loop system is unstable
SISO design for AFM – cont’d

Let’s focus on the dynamic requirements. We want the fastest transient with a bounded overshoot (≤ 10%).

In case of a reasonably high phase margin, we do not expect significant overshoots (due to the first order approximation of the complementary sensitivity). In turn, we might also consider the second order approximation, which relates the closed-loop overshoot with the phase margin in case of a second order approximation of the complementary sensitivity function.

\[ 10 \geq S_\% = 100e^{-\frac{\xi \pi}{\sqrt{1 - \xi^2}}} \quad \xi = \sin \left( \frac{\psi_m}{2} \right) \]

When solved, we get \( \xi \geq 0.59, \ \psi_m \geq 72^\circ \).
SISO design for AFM – cont’d

Based on the result we have just obtained, it is necessary to cross the 0 dB axis with unitary negative slope.

Hence we decide to put a zero in the dynamic part of the controller in order to change the slope from -2 to -1 before crossing.

A good position of such a (real) zero is exactly at the frequency $\omega_d$, i.e. after the bandwidth of the feedforward disturbance.

The corresponding controller will result like the following one

$$R_2 (s) = \frac{1 + s/\omega_d}{\text{some poles}} \quad L (s) = R_2 (s) \hat{L}_1 (s)$$
SISO design for AFM – cont’d

The corresponding Bode diagrams are as follows.

\[ \omega_c = 10 \text{ kHz}, \psi_m = 80^\circ \]

\[ |L(j\omega)| \]

\[ \hat{L}_1(j\omega) \]
SISO design for AFM – cont’d

The attenuation requirement on the measurement noise is no longer satisfied, we can introduced a pole within the dynamic part of the controller after the crossover frequency.

The corresponding controller will then look like the following one

\[
R_2 (s) = \frac{1 + \frac{s}{\omega_z}}{1 + \frac{s}{\omega_p}} \quad \hat{L} (s) = R_2 (s) \hat{L}_1 (s)
\]

The transfer function of the final controller is

\[
R (s) = R_1 (s) R_2 (s) = \frac{2.169e20s + 1.363e24}{s^3 + 6.283e05s^2}
\]
SISO design for AFM – cont’d

The corresponding Bode diagrams are as follows

\[ |L(j\omega)| \]
\[ |\hat{L}(j\omega)| \]

\[ \omega_c = 10 \text{ kHz}, \varphi_m = 75^\circ \]
SISO design for AFM – cont’d

Comments on the violation of $|L| \leq |L_1|$: just insert a lag compensator.
SISO design for AFM – cont’d

The Bode diagrams of the sensitivity function are as follows.
SISO design for AFM – cont’d

Step response of the closed-loop system (note the oscillations due to the presence of complex and conjugate poles).
SISO design for AFM – cont’d

The reason of these oscillations can be found on the Bode diagram of the loop transfer function.

The resonance peak is still close to the 0 dB axis.
SISO design for AFM – cont’d

We can try to fix this problem by designing a proper notch transfer function to filter the output of the controller we have selected.

A candidate solution could be the following one:

\[ N(s) = \frac{s^2 + 2\xi_1 \omega_1 s + \omega_1^2}{s^2 + 20\xi_1 \omega_1 s + \omega_1^2} \]

which “substitutes” the complex and conjugate poles of the original system with corresponding and better damped ones.
SISO design for AFM – cont’d

We can then compare the step responses of the two solutions with and without notch filter.