Transferring Value Functions via Variational Methods

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Problem

- The agent has solved a finite set of **source tasks** $\mathcal{M}_{\tau_1}, \mathcal{M}_{\tau_2}, \dots, \mathcal{M}_{\tau_M}$ sampled from some **distribution** $\mathcal{D}$
- Each task is an MDP $\mathcal{M}_\tau = \langle \mathcal{S}, \mathcal{A}, \mathcal{P}_\tau, \mathcal{R}_\tau, p_0 \rangle$
- A parametric approximation to their **optimal value functions** is available

$$\mathcal{W}_s = \{\boldsymbol{w}_1, \boldsymbol{w}_2, \dots, \boldsymbol{w}_M\} \text{ s.t. } Q_{\boldsymbol{w}_j} \simeq Q^*_{\tau_j}$$

- **Assumption**: all tasks share **similarities** in their optimal value functions [4]
- **Goal**: use this knowledge to speed-up the learning process of a new **target task** $\mathcal{M}_\tau$ sampled from $\mathcal{D}$

Motivation

- Reinforcement learning algorithms have enjoyed many success stories in complicated tasks
- High **sample complexity** remains a major issue
- Must **adapt** to changing environments and goals
- **Prior knowledge** from related tasks is often available in practice → **Transfer learning** [6]
- Need for transfer algorithms that are **general** and widely **applicable**

Contributions

1. **Algorithmic**. We propose a general **framework** for transferring value functions in RL and two **practical algorithms**
   - We learn a **prior** distribution over optimal Q-functions using the given source tasks
   - **Variational** approximation of the corresponding posterior for a new target task
   - Efficient **exploration** via posterior sampling
   - **Any** differentiable Q-function approximator and prior/posterior models could be used
2. **Theoretical**. We provide a theoretical analysis of our practical algorithms offering a better insight into their behavior
3. **Empirical**. We empirically evaluate our algorithms on four different domains with increasing level of difficulty

Variational Transfer Framework

<u>Idea</u>: use the source weights $\mathcal{W}_s$ to estimate the distribution $p(\boldsymbol{w})$ over optimal Q-functions induced by $\mathcal{D}$

- How to characterize $p(\boldsymbol{w}|D) \propto p(D|\boldsymbol{w})p(\boldsymbol{w})$ given a dataset D of N samples from the target task?
- **PAC-Bayes argument** [3]: the likelihood $p(D|\boldsymbol{w})$ decays exponentially as the TD error of $Q_{\boldsymbol{w}}$ on D increases

$$p(\boldsymbol{w}|D) \simeq \frac{e^{-\Lambda \|B_{\boldsymbol{w}}\|_D^2} p(\boldsymbol{w})}{\int e^{-\Lambda \|B_{\boldsymbol{w}'}\|_D^2} p(d\boldsymbol{w}')}$$

- **Problem**: computing the Gibbs posterior is often intractable → **Variational approximation** [1]

$$\min_{\boldsymbol{\xi} \in \Xi} \mathcal{L}(\boldsymbol{\xi}) = \mathbb{E}_{\boldsymbol{w} \sim q_{\boldsymbol{\xi}}}\left[\|B_{\boldsymbol{w}}\|_D^2\right] + \frac{\lambda}{N} KL\left(q_{\boldsymbol{\xi}}(\boldsymbol{w}) \,||\, p(\boldsymbol{w})\right)$$

Main Properties

1. **Prior estimation**: summarize the information to transfer into a single distribution and use it to guide the learning process of the target task
2. **Exploration via posterior sampling** [5, 2]: at each time, the agent guesses the solution of the target task according to the current posterior and acts accordingly
3. **Black-box optimization**: minimizing the variational objective requires only differentiability of the models involved

Algorithm Variational Transfer

```
Input: Target task M_τ, source weights W_s
  Estimate prior p(w) from W_s
  ξ ← argmin_ξ KL(q_ξ||p), D ← ∅
  repeat
    Sample initial state: s_0 ~ p_0
    while s_h is not terminal do
      a_h = argmax_a Q_w(s_h, a) for w ~ q_ξ(w)
      s_{h+1} ~ P_τ(·|s_h, a_h), r_{h+1} = R_τ(s_h, a_h)
      D ← D ∪ ⟨s_h, a_h, r_{h+1}, s_{h+1}⟩
      ξ ← optimizer(ξ, ∇_ξ L(ξ))
    end while
  until forever
```

Practical Algorithms

Gaussian Variational Transfer (GVT)

- Prior: $p(\boldsymbol{w}) = \mathcal{N}(\boldsymbol{\mu}_p, \boldsymbol{\Sigma}_p)$
- Posterior: $q_{\boldsymbol{\xi}}(\boldsymbol{w}) = \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

Mixture of Gaussian Variational Transfer (MGVT)

- Prior: $p(\boldsymbol{w}) = |\mathcal{W}_s|^{-1} \sum_{\boldsymbol{w}_s \in \mathcal{W}_s} \mathcal{N}(\boldsymbol{w}|\boldsymbol{w}_s, \sigma_p^2 \boldsymbol{I})$
- Posterior: $q_{\boldsymbol{\xi}}(\boldsymbol{w}) = C^{-1} \sum_{i=1,\dots,C} \mathcal{N}(\boldsymbol{w}|\boldsymbol{\mu}_i, \boldsymbol{\Sigma}_i)$

Finite-Sample Analysis

Bound the **expected Bellman error** under the optimal variational distribution for a dataset of N samples

$$\mathbb{E}_{q_{\widehat{\boldsymbol{\xi}}}}\left[\left\|\widetilde{B}_{\boldsymbol{w}}\right\|_\nu^2\right] \leq 2\left\|\widetilde{B}_{\boldsymbol{w}^*}\right\|_\nu^2 + \upsilon(\boldsymbol{w}^*) + c_1\sqrt{\frac{\log\frac{2}{\delta}}{N}} + \frac{c_2 + \lambda d \log N + \lambda\,\varphi(\mathcal{W}_s)}{N} + \frac{c_3}{N^2}$$

1. **Approximation error** due to the limited hypothesis space
2. **Variance** due to a biased estimation of the Bellman error
3. **Variance** due to the finite samples
4. **Likelihood** of the optimal target weights under the prior

GVT: Distance to the prior mean ☹

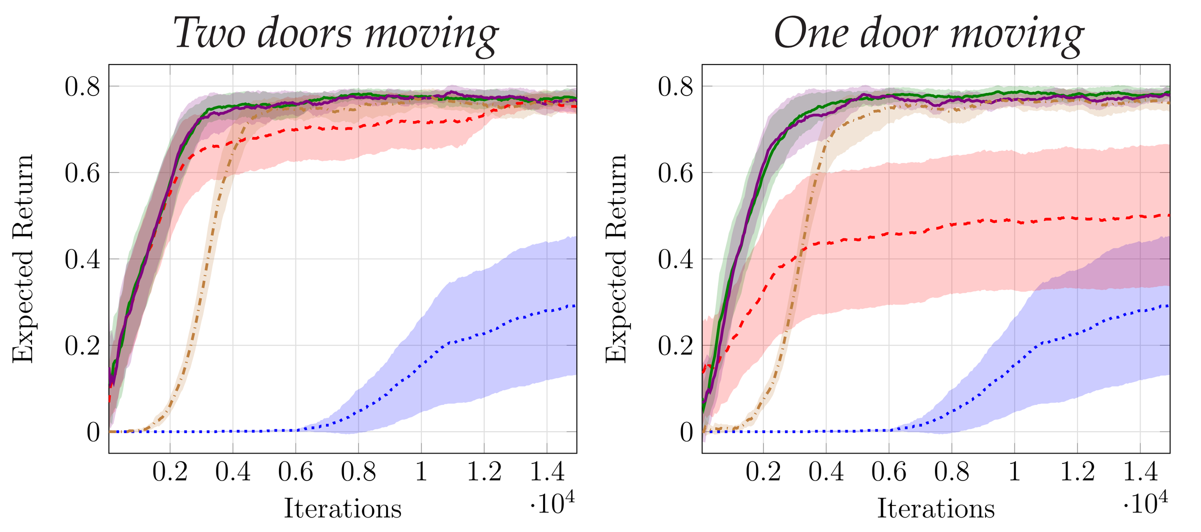
$$\varphi(\mathcal{W}_s) = \|\boldsymbol{w}^* - \boldsymbol{\mu}_p\|_{\boldsymbol{\Sigma}_p^{-1}}$$

MGVT: Softmin distance to the sources ☺

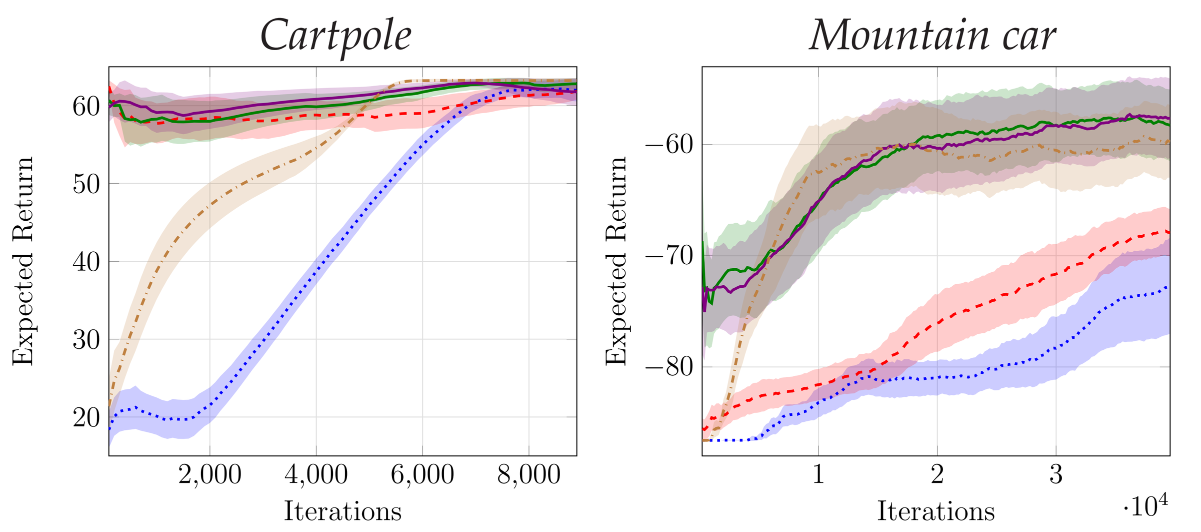
$$\varphi(\mathcal{W}_s) = \operatorname{softmin}_{\boldsymbol{w} \in \mathcal{W}_s}\left(\|\boldsymbol{w}^* - \boldsymbol{w}\|\right)$$

Empirical Results

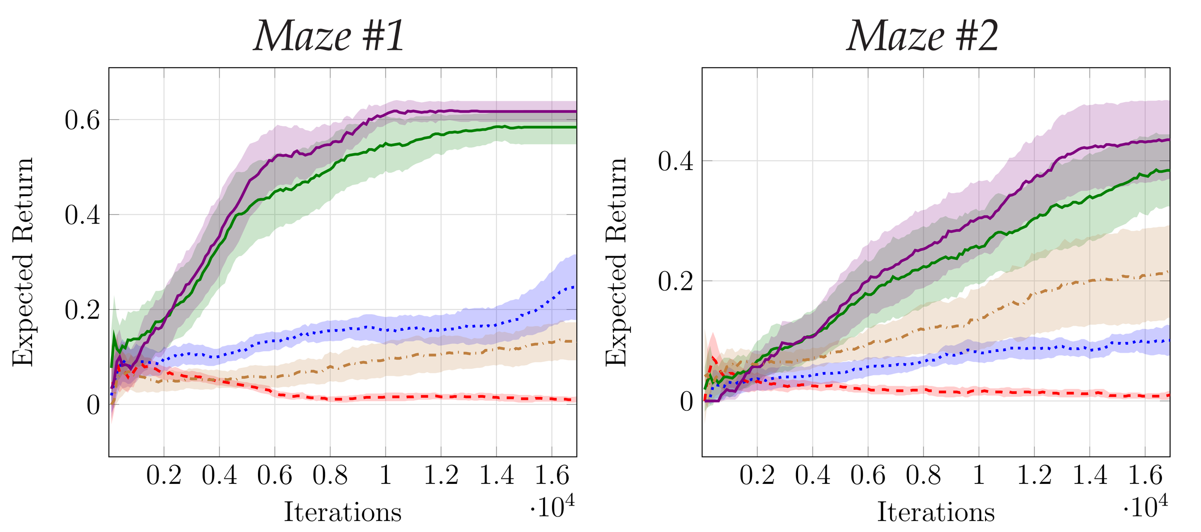
The Rooms Problem



Classic Control



Maze Navigation



- No transfer (DDQN)
- Fine-tuning from random source task
- GVT
- MGVT with 1 component
- MGVT with 3 components

References

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